AN EXPLICIT AND EFFICIENT SELF-DEVELOPED ALGORITHM FOR IBFSOF A TRANSPORTATION PROBLEM

Ojekudo*; Nathaniel Akpofure; Opara Jude*****

*(Ph.D),
Department of Computer Science,
Ignatius Ajuru University of Education Rivers State,
Port Harcourt, Rivers Nigeria
Email Id: nathojekudo@gmail.com

**Department of Mathematics and Statistics/Computer Science,
Ignatius Ajuru University of Education Rivers State,
Port Harcourt, Rivers Nigeria
Email id: judend88@yahoo.com

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ABSTRACT

The study was aimed at developing an explicit and efficient algorithm for tackling IBFS of a transportation problem. In this study, an efficient and explicit technique of obtaining an IBFS to a transportation problem was developed. The developed technique was named "One Least Cost Row Column Difference Method (OLCRCDM)", and the technique was practically demonstrated with ten problems which were numerical in nature. Five techniques which are in existence such as: NWCM, CMM, LCM, RMM, and VAM were compared with the proposed approach. Conclusively, the proposed OLCRCDM produced a better IBFS in the whole problems employed in the study, as it leads to optimal solution for about 70% of the employed numerical examples.

KEYWORDS: *Olcrcdm, Transportation Problem, Ibfs, Optimal Solution, Proposed Algorithm.*

INTRODUCTION

The Transportation Problem (TP) that was introduced by Hitchcock in 1941 is a recognized network optimization problem (Babu et al, 2014). To ascertain the optimal costs of transporting a commodity from different supply points to different demand points, such that the sum of the transportation cost is minimized; which is the primary objective of the TP. However, the parameters of the TP are the amounts accessible at the supply points, the amount needed at the demand points and the price of transporting one unit from a specific supply point to a specific demand point (Unit costs). A flow of products or materials between two companies' organizations can be regarded as a transportation, whose objective it to determine possible means of moving homogeneous materials or products from numerous sources to numerous destinations in order to minimize the total cost (Haruna, 2010).

A linear TP is a special category of linear programming problems in which a transportation technique can tackle (Chen et al, 2020). Since optimizing the objective function of a linear programming problem is the primary aim, hence; optimization of the linear objective function is tackled with linear programming, ensuring the linear constraints are kept (Charkhgard et al, 2018). There are several techniques of obtaining the IBFS for a balance TP (Juman& Hoque, 2015; Juman et al, 2013; Chhibber et al, 2021).The primary goal in a TP is to achieve the minimum cost of transportation via a transportation technique, which could be achievable after a feasible solution is being identified (Zhu et al, 2021; Bisht & Srivastava, 2020).

Hence, having known that the primary aim of a TP is to reduce the cost of transportation, which perhaps is obtained from the optimal solution, this study is focused at determining an efficient method of solving an initial basic feasible solution (IBFS) of a TP.

Transportation Problem in Linear Form

The process of distributing each item from sources to destinations, so that overall distribution costs are minimized is known as linear TP. Regardless of this, the volume of goods shipped per commodity is constant (Inyama, 2007). If W is the distribution cost for total and Z_{ij} is the quantity of units that is to be distributed from source i (g_i) to destination j (h_j) and K_{ij} is the unit transportation cost from source i to j, then the formulation of linear programming problem is:

$$\left.\begin{aligned} &Minimize\ W = \sum_{i=1}^{m}\sum_{j=1}^{n} K_{ij} Z_{ij} \\ &Subject\ to: \sum_{i=1}^{m} Z_{ij} = h_j,\ j = 1,2,\cdots,n \\ &\qquad \sum_{j=1}^{m} Z_{ij} = g_i, i = 1,2,\cdots,m \\ &\qquad where \quad Z_{ij} \geq 0 \end{aligned}\right\} \tag{1}$$

The two constraints in Equation (1) is said to be balance if

$$\sum_{i=1}^{m} g_i = \sum_{j=1}^{n} h_j$$

Otherwise, it is transformed to a standard TP before evaluation can be done. The sign of equality in the constraints makes of the equations to be redundant, which can be removed in such a way that the problem becomes $(m+n-1)$ constraints and $(m \times n)$ unknowns.

Linear Transportation Tableau

A unique tabular representation of the TPis known as a transportation tableau, which is represented in Table A.

TABLE A: LINEAR TRANSPORTATION TABLEAU

	Destinations						
Sources	1	2	....	j	....	n	$Supply\,(S_i)$
1	$\boxed{K_{11}}$ Z_{11}	$\boxed{K_{12}}$ Z_{12}	....	$\boxed{K_{1j}}$ Z_{1j}	....	$\boxed{K_{1n}}$ Z_{1n}	g_1
2	$\boxed{K_{21}}$ Z_{21}	$\boxed{K_{22}}$ Z_{22}	....	$\boxed{K_{2j}}$ Z_{2j}	....	$\boxed{K_{2n}}$ Z_{2n}	g_2

⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
i	$\boxed{K_{i1}}$ Z_{i1}	$\boxed{K_{i2}}$ Z_{i2}	....	$\boxed{K_{ij}}$ Z_{ij}	....	$\boxed{K_{in}}$ Z_{in}	g_i
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
m	$\boxed{K_{m1}}$ Z_{m1}	$\boxed{K_{m2}}$ Z_{m2}	....	$\boxed{K_{mj}}$ Z_{mj}	....	$\boxed{K_{mn}}$ Z_{mn}	g_m
$Demand\,(D_j)$	h_1	h_2	....	h_j	....	h_n	$\sum_{i=1}^{m} g_i = \sum_{j=1}^{n} h_j$

Various Techniques for Obtaining IBFS

In computing the optimal solution of a TP, the stage one to adopt is to obtain the IBFS. Hence, an arc flows which fulfils each demand requirement without moving more from any origin node than the available supply is known as the IBFS. The algorithms of some of the techniques employed to achieve the IBFS are stated here as:

i. NWCM – North-West Corner Method

ii. LCM – Least-Cost Method

iii. VAM – Vogel’s Approximation Method

iv. RMM – Row Minimum Method

v. CMM – Column Minimum Method

Algorithm for One Least Cost Row Column Difference Method (Self Developed)

A new and simple self-developed approach is used to obtain the IBFS of a TP and named **One Least Cost Row Column Difference Method (OLCRCDM)**. The new self-developed technique is employed through the following steps:

Step 1: Form a balance Transportation Table (TT) from a specified TP.

Step 2: Identify only one cell with the least unit transportation cost (K_{ij}) in the transportation tableau (breaking ties arbitrarily, by selecting the allocation cell value at the extreme left corner and at the topmost row if at least two cells occur in the same column). This means that, $Z_{ij} = Min.(g_i, h_j)$, which will either exhaust row i or column j.

Step 3: Locate the smallest and next smallest (shouldn’t be the same) costs of each row of the transportation tableau, and then determine the difference between these two values (penalties) for each row. Again, identify the highest and next highest (shouldn’t be the same) costs of each column of the transportation tableau, and then determine the difference between these two values (penalties) for each column. If the smallest and the next smallest are the same, ignore them throughout the iteration from using to compute the penalties in the particular row and identify the next two smallest values. Do the same in the case of column for highest and next highest

Step 4: Locate the row or column with the maximum penalty for the rows and columns (breaking ties arbitrarily by selecting the allocation cell value at the extreme left corner and at the topmost row if at least two cells occur in the same column). Locatethe cell with the lowest cost in the selected row or column and assign as many units as possible to it.

Step 5: Minimize the row supply and column demand by the number of units allotted to the cell and cross out the row supply or column demand that is satisfied, thereafter form a new Table.

Step 6: When a row and a column are both satisfied at the same time, one of them is crossed out only and the remaining row (or column) is assigned a zero supply (or demand). Any row or column with zero supply or demand should not be used in computing future penalties.

Step 7: Re-calculate the row and column penalties for the minimized (reduced) transportation tableau (as observed in step 3) and thereafter, move to step 4.

Step 8: This process is continued until all the requirements (supplies and demands) are satisfied, and the overall cost of the TP is computed using the relation:

$$W = \sum_{i=1}^{m} \sum_{j=1}^{n} K_{ij} Z_{ij} \qquad (2)$$

Numerical Problems

Ten numerical examples were employed in this study to illustrate the effectiveness of the developed technique. Example one was extracted from Okenwe (2018), example two was gotten from Osuji et al. (2014), example three was gotten from Abdul-Salam (2014), and examples five to seven were picked from a study ofOparaet al. (2016), while examples eight to ten were extracted from a study of Mollahet al. (2016).

Example 1

TABLE 1: INFORMATION FOR EXAMPLE 1

	MARKETS SEGMENTS			S
	P	Q	R	
Omo washing powder	5	4	6	13,000
Blue Band margarine	7	6	5	13,000
Vaseline	9	11	8	15,000
D	11,000	18,000	12,000	

The initial basic solution of Example 1 is obtained via the new algorithm as follows:

The smallest unit transportation cost is 4 and it falls at cell (1, 2), then we have;

$Z_{12} = Min.(g_1, h_2) = Min.(13, 18) = 13$

This means that OWP is fully satisfied and is crossed out while market two has 18 – 13 = 5 more units. We adjust the table to obtain.

	Markets			
Products	M_1	M_2	M_3	s_i
OWP	5 0	4 (13)	6 0	0
BBM	7	6	5	13
VAS	9	11	8	15
d_j	11	5	12	

Having done the crossing out for only one smallest unit transportation cost that satisfies the conditions, then; we then go to step three.

	Markets				
Products	M_1	M_2	M_3	s_i	Penalty
OWP	5 0	4 13	6 0	0	0
BBM	7	6	5	13	1
VAS	9	11	8	15	1
d_j	11	5	12		
Penalty	2	5	3		

The largest penalty is 5 and it corresponds to column 2 and the smallest cost in the same column is 6 and it is in cell (2, 2).

$Z_{22} = Min.(g_2, h_2) = Min.(13, 5) = 5$

This means that market $M_2$is fully satisfied and is crossed out while BBM has 13 – 5 = 8 more units. We adjust the table to obtain.

	Markets				
Products	M_1	M_2	M_3	s_i	Penalty
OWP	5 0	4 13	6 0	0	0
BBM	7	6 5	5	8	2
VAS	9	11 0	8	15	1
d_j	11	0	12		
Penalty	2	0	3		

The largest penalty is 3 and it corresponds to column 3 and the smallest cost in the same column is 5 and it is in cell (2, 3).

$Z_{23} = Min.(g_2, h_3) = Min.(8, 12) = 8$

This means that BBM is fully satisfied and is crossed out while market M_3 has 12 – 8 = 4 more units. We adjust the table to obtain.

	Markets				
Products	M_1	M_2	M_3	s_i	Penalty
OWP	5 0	4 13	6 0	0	0
BBM	7 0	6 5	5 8	0	0
VAS	9	11 0	8	15	1
d_j	11	0	4		
Penalty	-	0	-		

The largest penalty is 1 which corresponds to row 3 and the smallest cost in this row is 8 and it corresponds to cell (3, 3).

$Z_{33} = Min.(g_3, h_3) = Min.(15, 4) = 4$

This means that market M_3 is fully satisfied and is crossed out while VAS has 15 – 4 = 11 more units. We adjust the table to obtain.

	Markets				
Products	M_1	M_2	M_3	s_i	Penalty
OWP	5 0	4 13	6 0	0	0
BBM	7 0	6 5	5 8	0	0
VAS	9	11	8	11	-

		0	4		
d_j	11	0	0		
Penalty	-	0	-		

Since only one row/column is uncrossed out, the least cost method is employed to get the basic variable. This gives

$Z_{31} = Min.(g_3, h_1) = Min.(11, 11) = 11$

Hence, the final adjusted table is shown below as;

	Markets			
Products	M_1	M_2	M_3	s_i
OWP	5	4 13	6	13
BBM	7	6 5	5 8	13
VAS	9 11	11	8 4	15
d_j	11	18	12	

The basic feasible solution isZ_{12} = 13, Z_{22} = 5, Z_{23} = 8, Z_{31} = 11, Z_{33} = 4, which results in a transportation cost of

= (0, 13, 0, 0, 5, 8, 11, 0, 4), in Thousands.

$\bar{z} = (z_{11}, z_{B12}, z_{13}, z_{21}, z_{B22}, z_{B23}, z_{B31}, z_{32}, z_{B33})$

The total transportation cost is

13000(4) + 5000(6) + 8000(5) + 11000(9) + 4000(8) = ₦ 25300

Example 2

Example 2 is from a distributor Multi-Plan Limited that specializes in different types of drinks situated in Accra Ghana as displayed Table 1.

TABLE 1: INFORMATION OF EXAMPLE 2

	Markets Segments				S
	P	Q	R	V	
PR	15	10	4	20	15,000
OV	7	6	8	3	25,000
ML	1	9	5	3	10,000
D	20,000	10,000	8,000	12,000	

PR – P. Red, OV- Ovidio, ML - Merlot

Following the steps of the new algorithm in this study, it implies as follows:

The smallest unit transportation cost is 1 and it falls at cell (3, 1), then we have;

$Z_{31} = Min.(g_3, h_1) = Min.(10, 20) = 10$

This means that Merlot is fully satisfied and is crossed out while market A has 20 – 10 = 10 more units. We adjust the table to obtain.

	Markets				
Sources	**P**	**Q**	**R**	**V**	s_i
PR	15	10	4	20	15
OV	7	6	8	3	25
ML	1 10	9 0	5 0	3 0	0
d_j	10	10	8	12	

Having done the crossing out for only one smallest unit transportation cost that satisfies the conditions, we then go to step three.

	Markets					
Sources	**P**	**Q**	**R**	**V**	s_i	Penalty
PR	15	10	4	20	15	6
OV	7	6	8	3	25	3
ML	1 10	9 0	5 0	3 0	0	-
d_j	10	10	8	12		
Penalty	8	4	4	17		

The largest penalty is 17 and it corresponds to column 4 and the smallest cost in this column is 3 and it corresponds to cell (2, 4).

$$Z_{24} = Min.(g_2, h_4) = Min.(25, 12) = 12$$

This means that market D is fully satisfied and is crossed out while Ovidio has 25 – 12 = 13 more units. We adjust the table to obtain.

	Markets					
Sources	**P**	**Q**	**R**	**V**	s_i	Penalty
PR	15	10	4	20 0	15	6
OV	7	6	8	3 12	13	1
ML	1 10	9 0	5 0	3 0	0	0

d_j	10	10	8	0		
Penalty	8	4	4	0		

The largest penalty is 8 and it corresponds to column one and the smallest cost in the same column is 7 that correspond to cell (2, 1).

$Z_{21} = Min.(g_2, h_1) = Min.(13, 10) = 10$

This means that market A is fully satisfied and is crossed out while Ovidio has 13 – 10 = 3 more units. We adjust the table to obtain.

	Markets					
Sources	**P**	**Q**	**R**	**V**	s_i	Penalty
PR	15 0	10	4	20 0	15	6
OV	7 10	6	8	3 12	3	2
ML	1 10	9 0	5 0	3 0	0	0
d_j	0	10	8	0		
Penalty	0	4	4	0		

The largest penalty is 6 and it corresponds to row 1 and the smallest cost in this row is 4 and it corresponds to cell (1, 3).

$Z_{13} = Min.(g_1, h_3) = Min.(15, 8) = 8$

This means that market C is fully satisfied and is crossed out while P. Red has 15 – 8 = 7 more units. We adjust the table to obtain.

	Markets					
Sources	**P**	**Q**	**R**	**V**	s_i	Penalty
PR	15 0	10	4 8	20 0	7	-
OV	7 10	6	8 0	3 12	3	-
ML	1 10	9 0	5 0	3 0	0	0
d_j	0	10	0	0		
Penalty	0	4	0	0		

The only penalty leftis 4, that falls in column 2 with a minimum cost of 6, which corresponds to row 2. Thus; $Z_{22} = Min.(g_2, h_2) = Min.(3, 10) = 3$. Hence row 2 is fulfilled and crossed out, while column 2 remained 10 – 3 = 7. Only cell Z_{12} is left after adjustment with 7 as a demand/supply. This gives the Table;

	Markets				
Sources	**P**	**Q**	**R**	**V**	s_i
PR	15	10 7	4 8	20	15
OV	7 10	6 3	8	3 12	25
ML	1 10	9	5	3	10
d_j	20	10	8	12	

The table can be summarized in its final form as shown below;

	Markets					
Sources	**P**	**Q**	**R**	**V**	s_i	r_i
PR	15	10 7	4 8	20	15000	r_1
OV	7 10	6 3	8	3 12	25000	r_2
ML	1 10	9	5	3	10000	r_3
d_j	20000	10000	8000	12000		
t_j	t_1	t_2	t_3	t_4		

$$\bar{z} = (z_{11}, z_{B12}, z_{B13}, z_{14}, z_{B21}, z_{B22}, z_{23}, z_{B24}, z_{B31}, z_{32}, z_{33}, z_{34})$$

= (0, 7, 8, 0, 10, 3, 0, 12, 10, 0, 0, 0), in thousands.

The total transportation cost is

7000(10) + 8000(4) + 10000(7) + 3000(6) + 12000(3) + 10000(1) = ₦ 236,000

The remaining eight examples are solved via the algorithm in this study as illustrated in Example one and two, and their results are presented in Table 11, along with the results of the five heuristics techniques.

Example 3

Bottling company in Imo State, Owerri plant, Nigeria, a distributor of different categories of drinks [Fanta(F), Coke(C), Sprite(S)] in different market segments as indicated in the Table 3.

TABLE 3: DATA OF EXAMPLE 3

	MARKETS SEGMENTS					s_i
	Mbaise	Orlu	Aba	Umuahia	Afikpo	
F	14	8	11	12	8	11,000
C	12	10	7	15	11	17,000
S	10	9	14	13	15	11,000

d_j	6,000	7,000	9,000	10,000	7,000	

Example 4

Example 4 is from a distributor of Nigerian Bottling Company Ltd (Owerri plant) that specializes in production of beverage products of different kinds like Sprite, Coca-Cola, Schweppes, Fanta etc as displayed below.

TABLE 4: DATA OF EXAMPLE 4

	Markets Segments					Supply
	Aba	Umuahia	Nnewi	Orlu	Eket	
Sprite	13	9	10	11	9	13000
Coca-Cola	12	10	7	14	9	16000
Schweppes	14	11	15	12	15	11000
Fanta	12	16	13	8	14	15000
Demand	10000	10000	12000	14000	9000	

Example 5

This is a TP with 4 markets and warehouses as displayed in Table 5.

TABLE 5: INFORMATION FOR EXAMPLE 5

	Markets				**S**
WH	M_1	M_2	M_3	M_4	
$\mathbf{W_1}$	2	5	6	3	**6**
$\mathbf{W_2}$	9	6	2	1	**9**
$\mathbf{W_3}$	5	2	3	6	**7**
$\mathbf{W_4}$	7	7	2	4	**12**
Demand	**10**	**4**	**6**	**14**	**34**

Example 6

A company in Calabar that produces/sells the following products as demonstrated in Table 6.

TABLE 6: INFORMATION FOR EXAMPLE 6

	Enugu	Akwa-Ibom	Anambra	Rivers	Supply
BF	45	52	63	57	15500
CF	58	48	56	54	12000
PS	52	55	62	58	1440

					0
WO	65	48	44	54	1160 0
Dem and	1260 0	125 00	13000	154 00	5350 0

Example 7

This is a TP with 4 markets (MKs) and 3 warehouses (WHs) as displayed in Table 7.

TABLE 7: INFORMATION FOR EXAMPLE 7

	MKs				
WHs	M_1	M_2	M_3	M_4	Supply
W_1	5	7	9	6	12
W_2	6	7	10	5	14
W_3	7	6	8	1	10
D	10	6	8	12	36

Example 8

This is a TP with 3 factories (Fs) and 4 showrooms (SRs) as shown in Table 8.

TABLE 8: DATA OF EXAMPLE 8

	SRs				
Fs	P_1	P_2	P_3	P_4	
K_1	3	1	7	4	300
K_2	2	6	5	9	400
K_3	8	3	3	2	500
D	250	350	400	200	

Example 9

This is a TP with 4 sources (S) and destinations (D) as shown in Table 9.

TABLE 9: DATA OF EXAMPLE 9

	D				
S	P_1	P_2	P_3	P_4	S
V_1	7	5	9	11	30
V_2	4	3	8	6	25
V_3	3	8	10	5	20
V_4	2	6	7	3	15
D	30	30	20	10	

Example 10

This is a TP with 3 sources and destinations as displayed in Table 10.

TABLE 10: INFORMATION FOR EXAMPLE 10

	Destinations			S
Sources	D_1	D_2	D_3	
S_1	4	3	5	90

S_2	6	5	4	80
S_3	8	10	7	100
D	70	120	80	

Summary of Result

Having obtained the IBFS via the developed technique using tenproblem sets, the results obtained using the new technique (OLCRCDM) was compared with the results achieved with the existing techniques.

TABLE 11: RESULTS COMPARISON VIA DIFFERENT TECHNIQUES WITH THE SELF-DEVELOPED TECHNIQUE

	Expl 1	Expl2	Expl 3	Expl4	Expl 5	Expl6	Expl 7	Expl 8	Expl 9	Expl 10
	Initial Basic Feasible Solution									
NWCM	270,000	420,000	454,000	605,000	149	2,817,000	192	4400	540	1500
LCM	261,000	264,000	384,000	517,000	83	2,657,000	192	2900	435	1450
VAM	270,000	236,000	381,000	513,000	92	2,657,000	190	2850	470	1500
RMM	261,000	281,000	384,000	561,000	92	2,659,400	216	2850	470	1450
CMM	270,000	327,000	384,000	573,000	88	2,663,500	216	3600	435	1500
OLCRCDM	253,000	236,000	377,000	517,000	83	2,657,000	190	2850	420	1390
Optimal Solution	**253,000**	**236,000**	**377,000**	**509,000**	**83**	**2,655,600**	**190**	**2850**	**410**	**1390**

Table 11 shows the transportation cost for the ten numerical examples employed. It is noticed that the NWCM and CMM did not provide any optimal result as their initial basic results; LCM and RMM provided only one optimal result; VAM three optimal results; while the Proposed OLCRCDM provided seven optimal results. The three solved problems in which its IBFS are not optimal for the developed technique are also very close to optimal value. Hence, the developed One Least Cost Row Column Difference Method (OLCRCDM) provides comparatively a better IBFS which is either optimal or very close to optimal than the results obtained by the traditional algorithms considering the examples employed in the study.

CONCLUSION

It can be concluded that this self-developed algorithm produces an IBFS, which is close to optimal solution and even in some cases being optimal. Hence, future study should consider looking at other techniques which were not employed in this study and compare their results with the outcome of the developed technique. Again, unlike other techniques employed in this study whose programming codes had been written for easier computation, the study recommends a future work of attempting to write a programming code for the developed technique.

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