Asian Journal of Multidimensional Research
ISSN: 2278-4853 Vol. 12, Issue12, December2023SJIF 2022 = 8.179
A peer reviewed journal

A REVIEW ON AMBIGUOUS SET THEORY

Pawan Kumar Singh*

*Department of Statistics,
Central University of Rajasthan,
Ajmer, Rajasthan, INDIA
Email id: pawansinghupc@gmail.com;
DOI:10.5958/2278-4853.2023.00149.0

ABSTRACT

Lately, the accurate assessment of uncertainty in data featuring fuzzy attributes has become a significant challenge. To address this, various frameworks such as fuzzy sets and intuitionistic fuzzy sets theory have been extensively proposed. A particular challenge arises when computing the complement of true or false membership values, especially in situations involving indeterminacy. In response to this, the concept of ambiguous set (AS) has emerged as a recent addition. The discussion includes a real-world example that demonstrates how dealing with unconsciousness and ambiguity in human perception motivated the development of ambiguous set theory. Ultimately, the study delves into the definition of ambiguous sets, their mathematical representation, and associated concepts.

KEYWORDS:*Fuzzy Set; Intuitionistic Fuzzy Set; Ambiguous Set; Uncertainty.*

1.INTRODUCTION

Recently, addressing the precise evaluation of uncertainty and vagueness in attributes has become a pivotal concern for data science researchers. For uncertainty management in data, many contributions were made by the researchers [1–3].

Recently, Singh et al. [4] introduced the ambiguous set (AS) theory to address the issue of ambuity data. The AS emphasizes on the [5]:

"Development, perspective, and dispersion of ambiguity, and its association with various conceptual details and human perceptions".

The fundamental notion of an AS is to characterize each event with four member- ship degrees, viz., true membership degree (TMD), false membership degree (FMD), partially true membership degree (PTMD), and partially false membership degree (PFMD), all of which are considered dependent.

Singh and Bose [6] expanded the idea of Singh et. al. [4], and discussed many characteristics of the AS. This theory is also intended to be addressed to the ambiguity and unconsciousness of human perception [7]. Singh and Huang [8] proposed different formulas to define TMD, FMD, PTMD, and PFMD. Singh [9] presented a number of aggregation operators for integrating single-valued ambiguous numbers (SVANs). Singh [5] demonstrated the application of AS in decision-making. Singh and Huang [10] presented the concept of four-valued ambiguous logic, whose application was shown in designing ambiguous inference system. Singh and Huang [11] proposed an ambiguous edge detection method to identify the edges in computed tomography scans of coronavirus disease 2019 cases.

The author has dedicated this study to elucidating the fundamentals of AS and its associated ideas through an illustrative example.

The remaining sections of the article are outlined as follows: Section 2 offers preliminary insights into the AS. Section 3 discusses the diverse mathematical definitions of AS. Concluding remarks and future directions are presented in Section 4.

2 Ambiguous Set (AS)

This section outlines fundamental definitions of Fuzzy Set (FS) and Intuitionistic Fuzzy Set (IFS), followed by the AS.

Definition 1: (FS) [12]. Consider a fixed universe U = {g} for any event g. A fuzzy set A for g ∈ U is defined by:

$$A = \{g, \Phi t(g) \mid g \in U\} \quad (1)$$

Here, Φ_t: U → [0, 1] represents the true membership degree function for A, and $\Phi_t(g) \in [0, 1]$ is the membership degree of g ∈ U in A.

Definition 2: (IFS) [13]. For a fixed universe U = {g}, an interval fuzzy set I

for g ∈ U is defined by:

$I = \{g, \Phi t(g), \Phi f(g) \mid g \in U\}$ (2)

Here, $\Phi t(g)$:U→[0, 1] and $\Phi f(g)$: U→[0, 1] represent the true and false mem- bership degrees, respectively. Both $\Phi t(g)$ and $\Phi f(g)$ satisfy: $0 \leq \Phi t(g) + \Phi f(g) \leq 1$.

A hesitant membership degree $\Phi h(g)$ is incorporated in IFS *I*, expressed as:

$I = \{g, \Phi t(g), \Phi f(g), \Phi h(g) \mid g \in U\}$ (3)

Here, in Eq. 3, $\Phi t(g)$, $\Phi f(g)$, and $\Phi h(g)$ satisfy:

$$\Phi t(g) + \Phi f(g) + \Phi h(g) = 1 \quad (4)$$

Next, the definition of AS is presented in terms of four membership degrees.

Definition 4: (AS) [8]. Consider a fixed universe *U*. An ambiguity set *S* for

g ∈*U* is defined by:

$$S = \{g, \Psi t(g), \Psi f(g), \Psi ta(g), \Psi fa(g) \mid g \in U \quad (5)$$

Here, in Eq. 5, $\Psi t(g)$, $\Psi f(g)$, $\Psi ta(g)$, and $\Psi fa(g)$ satisfy: $0 \leq \Psi t(g)+\Psi f(g)+\Psi ta(g)+ \Psi fa(g) \leq 2$.

In Definition 2, the terms $\Psi t(g)$, $\Psi f(g)$, $\Psi ta(g)$, and $\Psi fa(g)$ are denoted as the true membership function (TMF), false membership function (FMF), partially true membership function (PTMF), and partially false membership function (PFMF), respectively. Together, these four functions are collectively referred to as ambiguous membership functions (AMFs). Singh and Huang [8] proposed four categories of AMFs: T1AMFs, T2AMFs, T3AMFs, and T4AMFs.

3 Related Definitions for the AS

This section introduces various mathematical definitions related to the AS.

Definition 5: (Ambiguousness) [8]. The assignment of membership degrees to the event g is termed as an ambiguousness operation. An ambifier ∅= (Ψt, Ψf, Ψta, Ψfa) is a 4-tuple of membership functions Ψt, Ψf, Ψta, Ψfa: [0, 1]. When applied to g, the ambifier∅ produces an AS S in U as:

S = {g, Ψt(g), Ψf (g), Ψta(g), Ψfa(g)} (6)

An AS defines a region in a 2-dimensional (2D) space based on ambiguous membership functions (AMFs), known as an ambiguous region (AR). Mathematically, it can be defined as:

Definition 6: (AR) [8]. For g ∈ U, an AR, denoted as SRN, is a convex polygonal region with vertices (X_1(g), 0), (X_2(g), 0), (0, Y_1(g)), and (0, Y_2(g)).

Fig. 1 illustrates the membership degrees for S1 and S2. The green shaded region in this figure represents the AR.

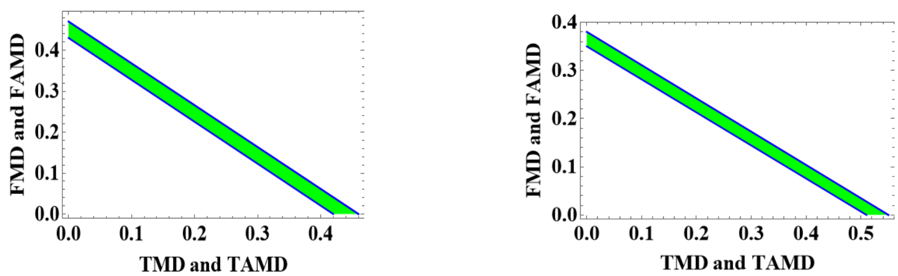

Fig. 1: Degree of memberships and the AR: (a) Membership degrees for S_1 (b) Membership degrees for S_2

The presence of ambiguity arises from unconsciousness. The quantification of this ambiguity can be measured using entropy, known as ambiguous entropy (AE). Mathematically, it can be defined as:

Definition 7: (AE). The AE for an AS *S* (Eq. 5) is defined as:

$$AE(S) = 1 - \frac{1}{4}[\Psi\mathrm{t(g)} + \Psi\mathrm{f(g)}] \times [\Psi\mathrm{ta(g)} - \Psi\mathrm{fa(g)}] \qquad (7)$$

Where, *S* is any event in the universe. Here, Ψt, Ψf, Ψta, Ψfa : [0, 1] for the ambiguity set *S*.

Definition 8: (AND operator) [7]. Two ambiguity setsS_1 and S_2can be combined with the AND operator, defined as:

S_{AND} = {min (Ψt_1, Ψt_2), max (Ψf_1, Ψf_2), max (Ψta_1, Ψta_2), max (Ψfa_1, Ψfa_2)} (8)

Definition 9: (OR operator) [7]. Two ambiguity setsS_1 and S_2 can be combined with the OR operator, defined as:

S_{OR} = {max (Ψt_1, Ψt_2), min (Ψf_1, Ψf_2), min (Ψta_1, Ψta_2), min (Ψfa_1, Ψfa_2)} (9)

In Eqs. 8 and 9, Ψt1 (g), Ψf 1(g), Ψta1 (g), and Ψfa1 (g) belong to S_1; whereasΨt2 (g), Ψf 2(g), Ψta2 (g), and Ψfa2 (g) belong to S_2.

By following Eqs. 8 and 9,S_1 and S_2 are obtained, satisfying the following properties:

P1: $(S_1)^C$= {g, Ψ1f (g), Ψt1 (g), 1 − Ψt1 (g), 1 − Ψf 1(g) | g ∈ U}, where

S_1 = {g, Ψt1 (g), Ψf 1(g), Ψta1 (g), Ψfa1 (g) | g ∈ U}.

Here, "*C*" denotes the complement operator.

P2: $S_1 \wedge S_2 = S_2 \wedge S_1$, and

P3: $S_1 \vee S_2 = S_2 \vee S_1$

4 Conclusions and Future Directions

In this study, related works on the AS theory was reviewed for the scientific community, providing a clear and detailed explanation. Various definitions and mathematical operations for the ASs were discussed in this article.

The AS theory can be applied in various domains, including digital image segmentation, time series prediction, decision-making model, and so on.

REFERENCES

1. Singh, P.: FQTSFM: A fuzzy-quantum time series forecasting model. Information Sciences 566, 57–79 (2021)

2. Singh, P.: A type-2 neutrosophic-entropy-fusion based multiple thresholding method for the brain tumor tissue structures segmentation. Applied Soft Com- puting 103, 107119 (2021)

3. Singh, P., Huang, Y.-P.: A four-way decision-making approach using interval- valued fuzzy sets, rough set and granular computing: a new approach in data classification and decision-making. Granular Computing 5, 397–409 (2020)

4. Singh, P., Huang, Y.-P., Lee, T.-T.: A novel ambiguous set theory to represent uncertainty and its application to brain MR image segmentation. In: Proc. of IEEE Int. Conf. on Systems, Man and Cybernetics (SMC), Bari, Italy, pp. 2460– 2465 (2019)

5. Singh, P.: An investigation of ambiguous sets and their application to decision- making from partial order to lattice ambiguous sets. Decision Analytics Journal 8, 100286 (2023)

6. Singh, P., Bose, S.S.: Ambiguous D-means fusion clustering algorithm based on ambiguous set theory: Special application in clustering of CT scan images of COVID-19. Knowledge-Based Systems 231, 107432 (2021)

7. Singh, P.: Ambiguous set theory: A new approach to deal with unconsciousness and ambiguousness of human perception. Journal of Neutrosophic and Fuzzy Systems 5(1), 52–58 (2023)

8. Singh, P., Huang, Y.-P.: Membership functions, set-theoretic operations, distance measurement methods based on ambiguous set theory: A solution to a decision- making problem in selecting the appropriate colleges. Int. J. Fuzzy Syst. 25, 1311–1326 (2023)

9. Singh, P.: A general model of ambiguous sets to a single-valued ambiguous numbers with aggregation operators. Decision Analytics Journal 8, 100260 (2023)

10. Singh, P., Huang, Y.-P.: A four-valued ambiguous logic: Application in designing ambiguous inference system for control systems. Int. J. Fuzzy Syst., 1–18 (2023) https://doi.org/10.1007/s40815-023-01582-2

11. Singh, P., Huang, Y.-P.: An ambiguous edge detection method for computed tomography scans of coronavirus disease 2019 cases. IEEE Trans. on Systems, Man, and Cybernetics: Systems, 1–13 (2023) https://doi.org/10.1109/TSMC. 2023.3307393

12. Zadeh, L.A.: Fuzzy sets. Information and control 8(3), 338–353 (1965)

13. Atanassov, K.T., Stoeva, S.: Intuitionistic fuzzy sets. Fuzzy sets and Systems

20(1), 87–96 (1986)